Measurement and Correlation of Turbulent Friction Factors of Thoria Suspensions at Elevated Temperatures

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When an aqueous flocculated suspension of thoria is pumped through a pipe with a sufficiently high velocity, shear-induced turbulence will develop just as for Newtonian fluids. The interparticle attractive force responsible for flocculation, which produces a shear-dependent viscosity in laminar flow, is overcome sufficiently by large shear-induced forces to permit the random eddying of turbulence to occur.

As with Newtonian fluids the turbulent behavior cannot be predicted theoretically, and recourse is made to phenomenological or empirical treatments. The Fanning friction factor-Reynolds number correlation for Newtonian fluids provides a useful framework for prediction of the turbulent flow pressure drops to be expected in pumping flocculated suspensions. The purpose of this paper is to extend the utility of this approach so that high temperature friction factors can be predicted from low temperature suspension properties. For this purpose a Reynolds number is defined containing a viscosity which can be readily extrapolated to elevated temperatures and which is phenomenologically meaningful.

BACKGROUND

Many measurements have been made of the turbulent friction factors when pumping various non-Newtonian fluids (*1* to *5*) at room temperatures. The major differences in the correlation techniques have been in the choice of a viscosity to use in the Reynolds number. The choice has depended on the particular rheological model to which the fluid conformed.

Fluids obeying the Ostwald-deWaele model

$$\tau = K\left(\frac{du}{dr}\right)^n \tag{1}$$

for example have been correlated with the generalized Reynolds number (*1*, *2*), whereas Bingham plastics, obeying the rheological equation

$$\tau = \eta\left(\frac{du}{dr}\right) + \tau_y \tag{2}$$

have been correlated with a Reynolds number containing a viscosity equal either to that of the suspending medium (*3*) or equal to the Bingham coefficient of rigidity (*4*, *5*, *6*). As a result of differences in the methods of computation of the viscosity from these rheological models the Reynolds number for a particular non-Newtonian material flowing turbulently in a pipe with a given velocity can vary by as much as a factor of 100.

CHOICE OF VISCOSITY

In order to place the correlation of all non-Newtonian turbulent fluid behavior on a rational basis it is proposed that two criteria should be met:

1. The choice of viscosity in the Reynolds number should be independent of the rheological model chosen to express the shear-dependent viscosity.
2. The choice of viscosity should be based on a reasonable phenomenological model of shear induced turbulence.

The only viscosity which meets both of these criteria is the wall shear stress viscosity (*7*). It is based on the laminar sublayer model of shear-induced turbulence which states that the viscosity is important only in the laminar region and therefore implies that it should be evaluated at the wall shear condition. The wall shear stress of a fluid flowing turbulently in a circular pipe can be calculated directly from pressure drop:

$$\tau_w = \frac{D\Delta P}{4L} \tag{3}$$

The viscosity appropriate to that shear stress can then be obtained from an experimental laminar flow shear diagram (plot of shear stress vs. shear rate) of the particular non-Newtonian fluid. In principle the shear diagram can be obtained experimentally with any viscometer type, providing the dimensions of the viscometer are such as to maintain laminar flow for the shear stress range of interest.

Alternately a rheological model can be assumed, the constants for the model determined experimentally over a suitable range, and the wall shear stress viscosity computed from the equation of the rheological model. Thus for example the wall shear stress viscosity of a Bingham plastic is

$$\mu_w = \frac{\eta}{1 - \tau_y/\tau_w} \tag{4}$$

whereas for an Ostwald-deWaele (power law) fluid the wall shear stress viscosity is

$$\mu_w = g_c\tau_w\,(K/\tau_w)^{1/n} \tag{5}$$

It should be noted here that the generalized Reynolds number method (*1*) of correlating the friction factor, although useful in eliminating a trial-and-error solution for the pressure drop, implicitly utilizes a viscosity which has no physical significance in turbulent flow. This viscosity, written in terms of the Ostwald-deWaele constants, is

$$\mu_a = g_cK\left(\frac{3n+1}{4n}\right)^n\left(\frac{8V}{D}\right)^{n-1} \tag{6}$$

In order for this expression to be physically meaningful the quantity $8V/D$ for a given fluid must be uniquely related to the wall shear rate. For turbulent flow this is not possible. The Rabinowitch-Mooney equation (8) which relates $8V/D$ to the wall shear rate is only meaningful for laminar flow. Thus the viscosity defined by Equation (6) is a contrived viscosity for turbulent flow useful for computation purposes only. Turbulent friction factor and heat transfer correlations with the generalized Reynolds number therefore should not be used for physical interpretations, since the Reynolds numbers will be considerably smaller than those calculated on the more rational basis of the wall shear stress.

APPROXIMATIONS

In the case of a Bingham plastic the Bingham constant η is a reasonable approximation of the wall shear stress viscosity at high shear stresses. This is presumably the basis for its proposed use by Hedstrom (6) and Weltmann (4) in their turbulent friction factor correlations, although they did not restrict its use to high shear stresses only. The use of η to predict friction factor is often a justified conservative approximation for Bingham plastics, particularly since friction factor is not highly sensitive to Reynolds number.

For pseudo plastics which have limiting viscosities at high shear stresses, such as the flocculated thoria suspensions used in this experiment, the high shear stress region can be represented by the Bingham plastic model and the same approximation applied. However for such materials it is necessary to insure that the shear diagram is obtained at sufficiently high shear stresses so that the limiting viscosity has in fact been reached. For flocculated thoria suspensions the limiting viscosity at high shear stress has been found equal to the Newtonian viscosity of a nonflocculated suspension of the same composition made by addition of a dispersing agent (9). This provides an independent, simple method of obtaining the limiting viscosity at high shear of thoria suspensions. It also provides a simple means of calculating the viscosity to use at elevated temperature, since the Newtonian (deflocculated) viscosity is directly proportional to the viscosity of the suspending medium (9, 10).

In the present study the experimental friction factors were correlated with a viscosity equal to the dispersed viscosity. This was measured at room temperature for each concentration and extrapolated to elevated temperatures as noted above. This approximation is accurate only for the high shear stress points. Deviations caused by the approximation at low shear stresses are in the direction of predicting too high a Reynolds number.

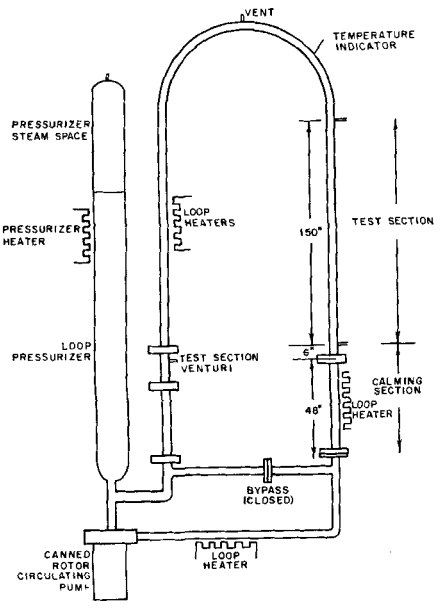

Fig. 1. Experimental loop schematic diagram.

APPARATUS AND PROCEDURE

The friction factors were measured in a specially constructed section which was installed as part of an existing high temperature circulating slurry loop (Figure 1). The section was fabricated from commercial 1½-in. sched-80 stainless steel pipe. A calming section containing a perforated cruciform baffle 36 pipe diameters long, and a straight section 4 diameters long preceded the test section. The test section itself consisted of 12.5 ft. of vertical loop piping. The differential pressure across the test section was sensed by a calibrated Swartwout differential pressure cell and continuously recorded.

The flow rate was measured downstream of the friction factor section with a calibrated Venturi meter. The differential pressure across the Venturi was sensed by a calibrated Swartwout differential pressure cell. The flow rate through the test section was controlled by regulating the speed of the loop circulating pump.

The slurry density in the test section was measured by stopping the circulating pump and reading the static differential pressure across the test section. To verify the density measurements slurry samples were removed from the loop during operation through a capillary tube sampler.

The experimental procedure consisted of charging the loop with thoria to the desired concentration, bringing the circulating suspension to temperature, and recording the test section pressure drop and flow rate for each pump speed. The pump was momentarily stopped at each temperature and the static pressure drop recorded. This static pressure drop was subtracted from the total pressure drop to determine the friction loss; it was also used to measure circulating concentration as explained above. At low flow rates the static pressure drop reading was observed frequently to insure that concentration was not changing owing to slurry dropout.

A water run was made prior to and following the thoria runs to establish the turbulent Newtonian friction factor correlation for the test section and to insure that there had been no change due to variation in pipe roughness.

Description of Thoria

Two thoria suspensions, designated as DT-12,13 and DT-4, were utilized in this experiment. They each consisted of granular particles of thorium oxide of approximately 1 μ mean diameter. The two suspensions were prepared by identical procedures (11) except that DT-12, 13 had been calcined at a maximum temperature of 1,600°C., while DT-4 had been calcined to a maximum of only 650°C. The higher calcining temperature results in a denser particle (12), so that at equal weight concentration the DT-4 suspensions contained higher volume fraction of solids. Pumping the suspensions at elevated temperatures tended to round off the edges of the particles. This increased the effective bulk particle density of each preparation, but the low-fired material was more affected. The weight mean particle size of the DT-4 preparation decreased slightly during the run from an initial value of 1.3 μ to a final value of 0.85 μ as determined by Andreasen pipette method.

Viscometer

The viscometer used was a vertical capillary-tube type utilizing a 0.018-in. tube 18 in. long. Flow was upward from a stirred pressurized autoclave through the capillary to a calibrated sight glass. The capillary was calibrated with water and with glycerol. Appropriate corrections for kinetic energy and static head were made.

Samples of the suspensions obtained during the course of the high temperature runs were chemically dispersed with sodium pyrophosphate and their Newtonian viscosities obtained at temperatures between 20° and 30°C. The results, plotted as specific viscosity vs. concentration, appear in Figure 2.

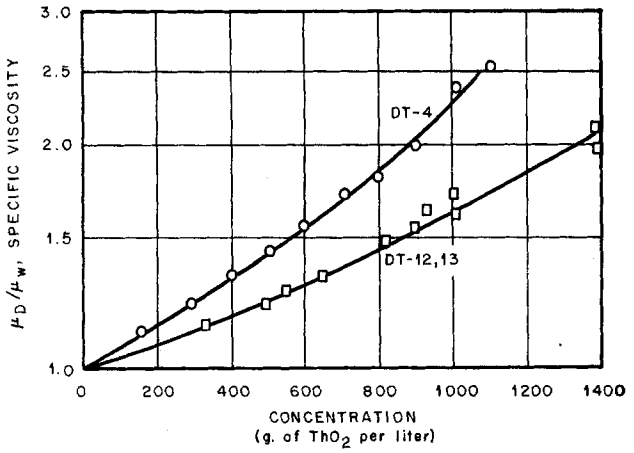

Fig. 2. Concentration dependence of specific viscosity.

In order to estimate the accuracy of the approximation of the wall shear stress viscosity by the dispersed viscosity, low temperature shear diagrams of the flocculated suspensions were also obtained with the capillary viscometer. The shear diagram for DT-12, 13 at 1,000 g. thorium dioxide/liter at 31°C. is shown in Figure 3 as an example along with the dispersed shear diagram. The smallest value of wall shear stress measured at this concentration in the turbulent friction factor study was 0.7 lb.$_f$/sq.ft. From the shear diagrams the viscosity of the flocculated suspensions at that shear stress is 0.001215 lb.$_m$/ft.-sec. compared with 0.00091 lb.$_m$/ft.-sec. for the dispersed suspension. Thus the Reynolds number obtained with the dispersed viscosity is in this case 33% greater than the one based on the wall shear stress viscosity. At the maximum shear stress obtained for this concentration, 2.25 lb.$_f$/sq. ft., the error is reduced to only 10%.

The errors cited above apply only to the room temperature data for the batch and concentration given. At lower concentrations for example, the maximum error is smaller since the suspension becomes more Newtonian. The error will in general cause an upward scatter of the friction factor data at all Reynolds numbers, since the temperature variation of the dispersed viscosiy shifts the Reynolds number for a given shear stress. No data are available on the effect of high temperature on Bingham yield stress of these suspensions. In the range 30° to 80°C. however it has been found (9) that the yield stress remains essentially constant. A quantitative estimate of

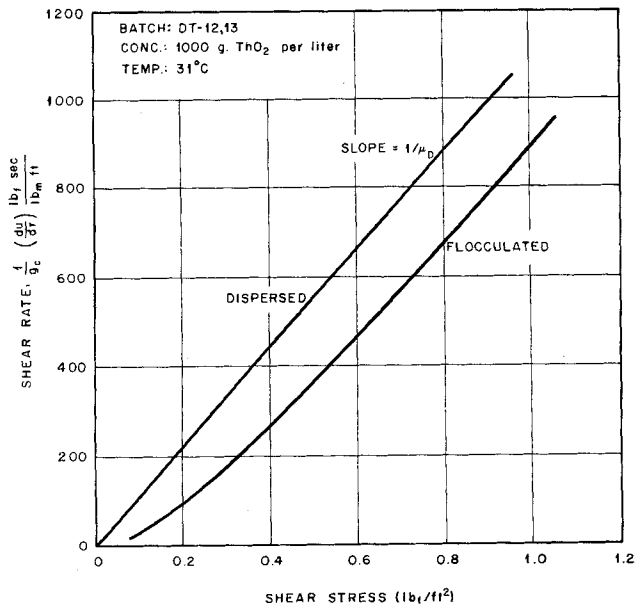

Fig. 3. Shear diagram of flocculated and dispersed suspensions.

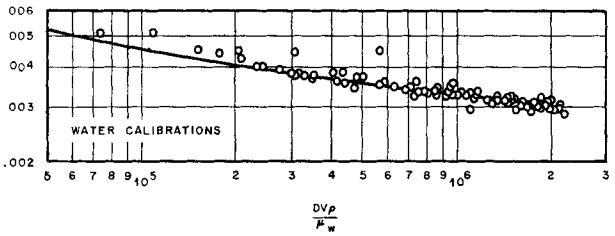

Fig. 4. Water friction factor plot. Temperature range 22° to 240°C. Velocity range 5 to 27 ft/sec.

the magnitude of the friction factor decrease for the high concentration suspensions would be a line which represents the lower envelope of the data rather than the mean value, since these are the higher shear stress points.

EXPERIMENTAL RESULTS

The experimental results were obtained from a series of loop runs at selected concentrations for each thoria preparation. The temperature of the loop was varied during a run between the ambient temperature (22° to 31°C.) and a maximum of 240°C. At each loop temperature a series of pressure drops were obtained for flow rates in the range 50 to 150 gal./min. equivalent to test section velocities of 10 to 27 ft./sec.

Water calibration tests were carried out at the start and the completion of the series of runs. The results of the water calibration tests showed no significant change during the run. A slight decrease in the friction factors at the highest temperature (highest Reynolds number) may have been due to a change in the surface roughness. The decrease also appears in the high temperature suspension runs. The water calibration results were combined in one graph and appear in Figure 4. The solid line, representing the predicted line for a surface roughness of 120 μin. (13), was used to compare these results with the slurry results. This roughness is considered reasonable for commercial stainless steel pipe.

Data were obtained for slurry preparation DT-12,13 in the concentration range 400 to 1,415 g. of thorium dioxide/liter. Typical results are shown in Figure 5. The friction factors obtained at concentrations below 800 g. of thorium dioxide/liter for this slurry showed no deviation

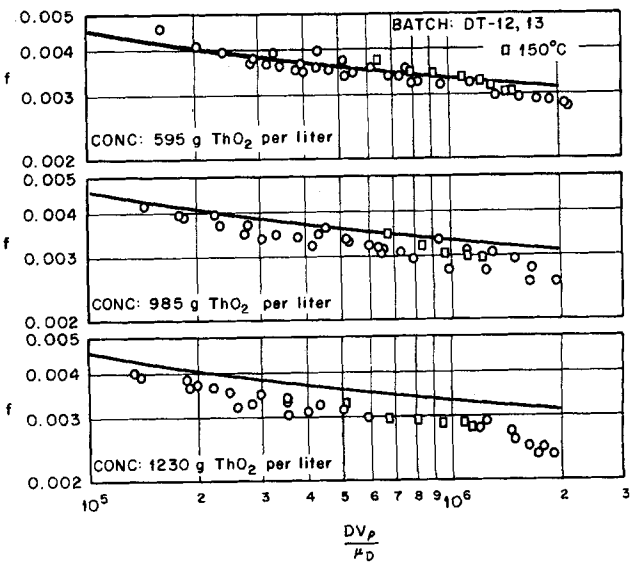

Fig. 5. Suspension friction factor plot. Temperature range 34° to 228°C. Velocity range 10 to 25 ft/sec.

from the water calibration, except as noted at high temperatures. At higher concentrations a decrease in friction factor at all Reynolds numbers occurs.

Friction factors for preparation DT-4 were obtained in the concentration range 205 to 575 g. thorium dioxide/liter. The results, as illustrated in Figure 6, are similar except that the decrease in friction factors occurs at a lower concentration. This is related to the difference in bulk particle density between these two preparations.

In each figure the data for the runs at 150°C. are shown separately. The Reynolds number variation for these data points is due to velocity change only. If the wall shear stress viscosity had been used instead of the dispersed viscosity, the Reynolds number would also have varied owing to viscosity change and the lower Reynolds numbers would be shifted to the left.

The scatter in the water data is low considering the use of remote-reading instruments. The decrease in friction factor at the higher concentrations is outside of the range of scatter of the suspension data. The greater scatter of the suspension data is due partially to the error introduced by the dispersed viscosity approximation.

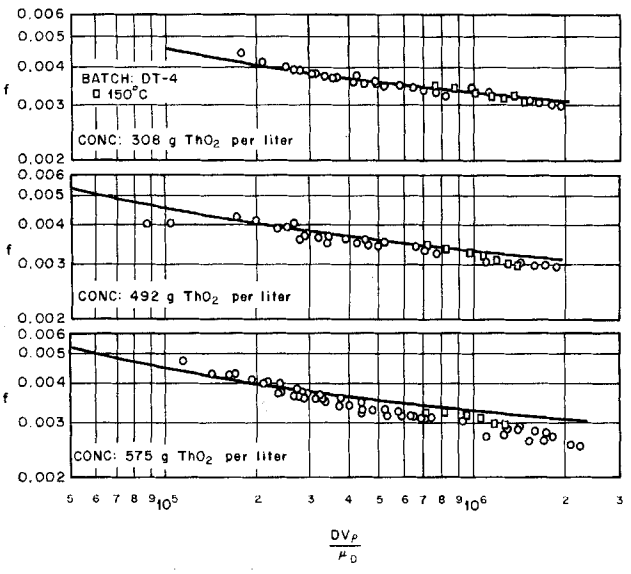

Fig. 6. Suspension friction factor plot. Temperature range 30° to 240°C. Velocity range 12.2 to 27 ft/sec.

DISCUSSION AND CONCLUSIONS

The use of the experimental and calculated values of the dispersed viscosity in the Reynolds number correlates the thoria suspension friction factors at all temperatures along lines coincident with or parallel to the water calibration line. Thus this procedure appears adequate for extrapolating room temperature experimental data to elevated temperatures for suspensions in which the shear stresses generated in turbulent flow are sufficiently high to permit the use of the dispersed viscosity approximation. In the case of the high concentration suspensions, for which the friction factors lie below but parallel to the water calibration, the procedure for estimating high temperature friction factors would be to obtain experimental room temperature turbulent friction factors to determine the extent of the friction factor depression. The data indicate that this depression does not change markedly with increased temperature, since the friction factor lines at high concentration are essentially parallel to the Newtonian curve.

The error introduced into the correlation curves by the use of the dispersed viscosity when the shear stresses are not sufficiently high is in the direction of increasing the Reynolds number above that calculated based on the actual wall shear stress viscosity. This in turn appears on the correlation curves as an upward bias of the low shear stress friction factors. Since the friction factors at high suspension concentrations even with this bias lie below the Newtonian curve, the cause for this shift downward cannot be attributed to the dispersed viscosity approximation.

The lowering of the turbulent friction factor curve with increasing non-Newtonian behavior has been reported in previous investigations (*1*, *2*) of pseudo plastic fluids. Most of this decrease in both cases is attributable to the use of the contrived generalized viscosity, which, as was noted, is much larger than the wall viscosity. However Bogue (*7*) replotted the data of Dodge and Metzner using the wall viscosity and found that the friction factors still fell below the Newtonian line at high concentration of polymer. Similar results also follow from replotting Shaver's friction factors (*2*); thus lowering of the friction factor appears to be a general property of highly non-Newtonian fluids.

Phenomenological Analysis of Friction Factor Decrease

The fundamental assumption inherent in the choice of the wall shear stress viscosity for the Reynolds number is that the laminar non-Newtonian effects expressed by the viscosity are confined to the wall region. However it is logical to assume that the same elements responsible for the non-Newtonian laminar behavior also cause a damping of the random eddying of turbulence and that this effect will reduce the momentum contained in the eddies compared with a Newtonian fluid with the same wall friction velocity. If a large reduction in momentum contained in the eddies occurs, an alteration in the velocity profile must occur since in the limit the disappearance of eddies will result in a laminar velocity profile. Thus if a change of profile results from eddy suppression, it will be toward a more laminar (less blunt) profile. This result was obtained experimentally by Shaver (*2*) for several pseudo plastic polymer solutions. Although the magnitude of the turbulent friction factor depression and the accompanying change in velocity profile may have been influenced by the fact that at least one of the fluids used by Shaver exhibited elastic properties, the relationship between the friction factor depression and the velocity profile change should not have been affected. The influence of the elasticity, if any, would have been to provide a more effective mechanism for damping eddies. The resulting large friction factor depression and velocity profile change are then both the result of the increased damping.

Measurements by Bogue (*7*) of the turbulent friction factors and turbulent velocity profiles of pseudo plastic (but not elastic) polymers and of clay suspensions showed roughly the same amount of velocity profile change as Shaver when compared with Shaver's data at the same friction-factor depression. Velocity profiles in agreement with those of Shaver were also obtained with thoria suspensions when compared at the same friction factor depression (*15*).

The prediction of the friction factor depression to be exhibited for a given non-Newtonian fluid, when correlated with the wall shear stress viscosity, is beyond the scope of this paper. It is clear however that the turbulent flow behavior of many non-Newtonian fluids cannot be predicted precisely on the basis of laminar flow measurements alone. Turbulence parameters, obtained either empirically from turbulent friction factors or velocity profiles or by the more fundamental techniques used to

characterize visco-elastic polymers (*16*), appear to be necessary to define turbulent behavior of suspensions of interacting particles.

NOTATION

D = pipe inside diameter, ft.
du/dr = shear rate
g_c = conversion constant, 32.2 lb.$_m$ ft./lb.$_f$ sec.^{2}
K = Ostwald-deWaele coefficient, lb.$_f$/sq.ft.-sec.^{n}
L = axial distance, ft.
n = Ostwald-deWaele exponent

Greek Letters

ΔP = pressure drop, lb.$_f$/sq.ft.
η = Bingham plastic coefficient of rigidity, lb.$_m$/ft.-sec.
μ = wall shear stress viscosity, lb.$_m$/ft.-sec.
μ_D = viscosity of dispersed suspension, lb.$_m$/ft.-sec.
μ_s = specific viscosity, ratio of dispersed viscosity to water viscosity at the same temperature
μ_w = water viscosity, (lb.$_m$/ft.-sec.)
ρ = density of suspension, lb.$_m$/cu. ft.
τ = shear stress, lb.$_f$/sq.ft.
τ_w = wall shear stress $D\Delta P/4L$, lb.$_f$/sq.ft.
τ_y = Bingham plastic yield stress, lb.$_f$/sq.ft.

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Thermodynamic Comparison of Large-Scale Liquefaction of Air, Hydrogen, and Helium

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In the United States in the 1940's, liquefaction of air was achieved on the large scale of millions of standard cubic feet per hour or thousands of tons per day of liquid products. This consisted of production primarily of liquid oxygen and to a lesser extent of liquid nitrogen and liquid argon. These were stored, transported, and used in some cases as liquid.

Two decades later in the United States, in the 1960's, hydrogen will be liquefied on the large scale of millions of standard cubic feet per hour. This is hundreds of tons per day of liquid hydrogen. This liquid hydrogen will be burned with liquid oxygen in National Aeronautics and Space Administration rockets for the exploration of space.

Oxygen and hydrogen are among the most reactive of chemical substances and are unlimited in their availability. This is not true of helium. However, it does not seem unreasonable to guess that re-liquefaction of helium, primarily for deep refrigeration purposes, will also be carried out on the large scale of millions of standard cubic feet per hour in the United States in the 1980's. This is hundreds of tons per day of helium liquefaction.

Efficient large-scale liquefaction of air or hydrogen or helium requires the expenditure of 2 to 4 kw. per 100 std. cu. ft./hr., and so a production level in the United States of millions of standard cubic feet per hour of any of these liquefied gases requires the expenditure of tens of thousands of kilowatts. The investment in gas compressors is millions of dollars, and the annual power cost is millions of dollars for such a scale of liquefaction.

This paper examines the problem of minimizing the power consumption for liquefaction and thus reducing the high cost of power, of compressors, and of other plant investment.

LOGARITHMIC REPRESENTATION

When certain physical properties of various fluids are represented on a logarithmic graph of temperature vs. pressure, or on a graph of the property vs. log pressure or log temperature and with families of curves at similar reduced temperature or pressure, the curves are of similar shape for the different fluids but are at different locations on the graph. Logarithmic scales of temperature and